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TEST PARTICLE MOTION AROUND BRANY BLACK HOLE IMMERSED IN AN EXTERNAL ASYMPTOTICALLY UNIFORM MAGNETIC FIELD

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Abstract

We investigate circular motion of charged and neutral particles around non-rotating black hole immersed in an external asymptotically uniform magnetic field. The effects of braneworlds on innermost circular stable orbits have been considered. Shown that innermost circular orbits (ISCO) decreases in decreasence of both brane charge and particle charge. Moreover, we have investigated energy extraction from black holes in braneworld through collision of two particles. Obtained that the presence of the brane charge parameter causes to decrease of the value of center of mass energy of colliding two charged particles, it means the brane charge acts as an additional gravity.

Keywords: black hole, braneworld, magnetic field, charged particle.

Physics and Astronomy Classification Scheme: 04.50.-h, 04.40.Dg.

Introduction

One way to study of properties of spacetime around black holes is to consider particle motion around the black hole. Generally, it is always astrophysically important to study motion of neutral [1], charged [2], magnetized [3, 4] and spinning [5] particles around the black holes in the presence of external magnetic fields. Today, it is the believable fact that all the black holes in the Universe are located in the magnetic field [6] and shown that the value of magnetic field in the vicinity of stellar black holes in order of \(10^8\)G, and in the vicinity of Supermassive black holes it is in order of \(10^4\)G. The vacuum solution of the Maxwell equation around the black hole has been first investigated by Wald in [7] and extended by several authors.

Another much interesting fact that our Universe is might be embedded in high dimensional spacetimes. One of the high dimensional gravities is braneworld have been first suggested by [8]. The braneworld’s static and spherically symmetric solution has been obtained by [9], which is mathematically similar to Reissner-Nordström metric and the several unpredictable properties of the spacetime has been studied by several authors [10, 11] in different topics for example, recently we have investigated the effects of the braneworld on neutron star plasma magnetosphere in our previously papers [12, 13].

The study of energy extraction mechanisms from black holes is always has been a hot topic in theoretical relativistic astrophysics. Because it may help to explain
the mechanisms of the existence of phenomenons like a jet in the center of active nuclei galaxies and source of ultra-high energy cosmic ray components. One of the models of the energy extraction models is particle collision around in the vicinity of the black hole. Several authors (see for example [15, 16, 4]) have studied the effects of the black hole rotation parameter and charge on the energy decay process. The study of particle motions around black holes with several energetic conditions is interesting and most actual because energy extractions from black holes are release by the particles. Above presented facts is good motivation for study test particle’s motion around a black hole.

In this work, we plan to investigate neutral and charged particles motion. In Sec.1 we have studied neutral particle motion around the black hole in braneworld and Paczyński-Wiita (pseudo-Newtonian) potential for the neutral particles. The Sec. 3 is dedicated to studying the motion of charged particle around the black hole in the presence of an external asymptotic uniform magnetic field. Finally, we have investigated energy extraction from the black hole due to the collision of two particles in Sec.collision. Throughout this work we use \((-, +, +, +)\) for the space-time signature and system of units where \(G = c = 1\). Latin indices run from 1 to 3 and Greek ones from 0 to 3.

1 Massive neutral particle motion

In this section, we will study a massive electrically uncharged particle motion around a black hole in braneworlds. The spacetime metric of a non-rotating black hole in braneworlds in spherical coordinates takes the following form [9]

\[
ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)
\]

with lapse function

\[
f = 1 - \frac{2M}{r} + \frac{Q}{r^2}.
\]

Here \(M\) is total mass of the black hole, \(Q\) is the brane charge (some literatures called tidal charge but originally Weil charge) parameter. The horizon of the black hole in braneworld can be found as using the standard condition \((g_{tt} = 0)\),

\[
r_h = M \left(1 \pm \sqrt{1 - \frac{Q}{M^2}}\right) \leq 2M.
\]

One can see from equation (2) there are two horizons around the black hole in the presence of brane charge: inner and outer horizons. However inner horizon is not physically, because we cannot reach its surface.

In this work, we have tested the effects of positive values of the brane charge \(Q > 0\). In order to get the equations of the motion we consider Lagrangian density for a neutral particle with mass \(m\) which can be written as

\[
\mathcal{L} = \frac{1}{2} m g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu
\]
The conserved quantities of motion, such as the energy and the angular momentum of the particle can be found in the following form

\[ p_t = \frac{\partial L}{\partial \dot{t}} \implies g_{tt} \dot{t} = -E \]

\[ p_\phi = \frac{\partial L}{\partial \dot{\phi}} \implies g_{\phi\phi} \dot{\phi} = l \]

(4)

where \( E/m = \mathcal{E}, L/m = l \) taking into account equation (4) the Lagrangian can be written as

\[ 2\mathcal{L} = g_{tt}\dot{t}^2 + g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 + g_{\phi\phi}\dot{\phi}^2 = \epsilon \]

(5)

where \( \epsilon = 0 \) for null geodesics, \( \epsilon = 1 \) is for time-like geodesics, \( \epsilon = -1 \) is for space-like geodesics. One can easily calculate the equations of neutral massive particle motion in the following form:

\[ \dot{r} = \sqrt{\mathcal{E}^2 + g_{tt}\left(1 + \frac{\mathcal{K}}{r^2}\right)} \]

(6)

\[ \dot{\theta} = \frac{1}{g_{\theta\theta}} \sqrt{\mathcal{K} - \frac{l^2}{\sin^2 \theta}} \]

(7)

\[ \dot{\phi} = \frac{l}{g_{\phi\phi}} \]

(8)

\[ \dot{t} = -\frac{\mathcal{E}}{g_{tt}} \]

(9)

where \( \mathcal{K} \) is the Carter constant. Here we will consider motion of the particle in the equatorial plane in which \( \theta = \pi/2 \) and \( \dot{\theta} = 0 \) in the case the Carter constant takes the form \( \mathcal{K} = l^2 \) and using the fact the equation of the motion takes the following standard form

\[ \dot{r}^2 = \mathcal{E}^2 - 1 - 2V_{eff} \]

(10)

where the last

\[ V_{eff} = \frac{1}{2} \left[ f \left(1 + \frac{l^2}{r^2}\right) - 1 \right] \]

(11)

the effective potential of the neutral particle.

1.1 Innermost stable circular orbits of neutral particle

To derive the circular motion of the neutral particle motion around the black hole in braneworlds, we will use the standard condition for circular motion \( \dot{r} = 0 \) (no radial motion) and \( \ddot{r} = 0 \) (no forces on the circular orbits) that allows obtaining expressions for the energy and the momentum of the particle. The critic angular momentum for circular stable orbits
\[ L^2 = \frac{r^2(Q - Mr)}{r(r - 3M) + 2Q}, \quad (12) \]

and the energy of the neutral particles at the circular stable orbits

\[ E^2 = \frac{(r(r - 2M) + Q)^2}{r^2(r(r - 3M) + 2Q)}. \quad (13) \]

The expression of the ISCO radius for the neutral particles around black hole in braneworlds can be found

\[ r_{isco} = 2M + \frac{\frac{3}{M} \sqrt{8M^6 - 9M^4Q + 2M^2Q^2 + M^2Q\sqrt{5M^4 - 9M^2Q + 4Q^2}}}{M (4M^2 - 3Q)} + \frac{M}{\sqrt{8M^6 - 9M^4Q + 2M^2Q^2 + M^2Q\sqrt{5M^4 - 9M^2Q + 4Q^2}}}, \quad (14) \]

it is at \( Q = 0 \) (pure the Schwarzschild black hole case) each part of the equation (14) equals to \( 2M \) and the ISCO radius equals \( r_{isco} = 6M \).

Figure 1: The ISCO radius of the neutral particle as a function of brane charge \( Q \)

We demonstrated radius of innermost circular orbits as a function of the brane charge parameter. One can see that the ISCO come closer to that the central object in the presence of the positive brane charge due to compressive effect of high dimensions.

2 pseudo-Newtonian potential

Here we will derive pseudo-Newtonian potential [14] for the black hole in the braneworlds which is an interesting astrophysical object. First, we will calculate Keplerian angular momentum to derive PW potential

\[ \Omega_K = \frac{L^2}{E^2} = \frac{r(Q - Mr)}{(r(r - 2M) + Q)^2}. \quad (15) \]
The general form of the pseudo-Newtonian potential

\[ V_{PW} = \int F_{CF} dr, \quad F_{CF} = \frac{\Omega_K}{r^3} \]  

(16)

Here \( F_{CF} \) is centrifugal forces. Taking into consideration (15), one can easily calculate the PW potential as

\[ V_{PW} = -\frac{2Mr - Q}{2(-2Mr + Q + r^2)} \]  

(17)

The equation (17) at \( Q = 0 \) the potential becomes to the Schwarzschild one

\[ V_{PW} = -\frac{M}{r - 2M} \]  

(18)

We plot the radial profile of the PW potential to show the effects of the brane charge \( Q \).

Figure 2 illustrated the radial profile of the PW potential for the different values of the brane charge \( Q \). One can see from the figure that the presence of the brane charge \( Q \) increases the value of the PW potential.

### 3 Charged particle motion

In this subsection we will study charged particle motion around black hole in braneworlds in the presence of the external uniform magnetic field. Four potentials of electromagnetic field around the black hole in braneworld have been found in [17], in non-rotating case \( (a = 0) \) the expression of the four-potentials become the following form

\[ A_{\mu} = \frac{1}{2} Br^2 \sin^2 \theta(0, 0, 0, 1) \]  

(19)

where \( B \) is the value of asymptotic uniform magnetic field around the black hole. In contrast to this work, we have considered that the bran charge cannot create
an electric field around the black hole \( A_0 = 0 \) because the bran charge has no electromagnetic properties. The Lagrangian for the charged particle with mass \( m \) and electric charge \( e \) in magnetic field

\[
\mathcal{L} = mg_{\mu\nu}u^\mu u^\nu + eu^\mu A_\mu
\]  

(20)

The conserved quantities, the energy and the angular momentum can be found by

\[
p_t = \frac{\partial \mathcal{L}}{\partial \dot{t}} = mg_\mu \dot{t},
\]

(21)

\[
p_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = mg_\phi \dot{\phi} + eA_\phi,
\]

(22)

Taking into consideration the equations (19-22), the equation of motion of charged particles in equatorial plane \( \theta = \pi/2 \) can be expresses as

\[
\dot{t} = \frac{\mathcal{E}}{f(r)},
\]

\[
\dot{r} = \sqrt{\mathcal{E}^2 - f(r) \left[ 1 + \left( \frac{l}{r} - \frac{1}{2} \omega_B r \right)^2 \right]},
\]

\[
\dot{\phi} = \frac{l}{r^2} - \frac{1}{2} \omega_B
\]

(23)

Now will calculate effective potential for a charged particle around the black hole immersed in the uniform magnetic field in branewold using Hamilton-Jacobi (H-J) equation. The H-J equation for test charged particle with mass \( m \) and the charge \( q \) can be expressed as

\[
g_{\mu\nu} \left( \frac{\partial S}{\partial x^\mu} + qA_\mu \right) \left( \frac{\partial S}{\partial x^\nu} + qA_\nu \right) = -m^2,
\]

(24)

the solution of equation (24) can be sought in the following form

\[
S = -\mathcal{E} t + \mathcal{L} \phi + S_r(r),
\]

(25)

where conservative quantities \( \mathcal{E} \) and \( \mathcal{L} \) are the energy and the angular the momentum of the test particle respectively. Equations of motion of the charged particles using conservative quantities can be written as

It is convenient to consider particle motion on equatorial plane \( \theta = \pi/2, \ (\dot{\theta} = 0) \) where the equation of motion for radial part reduces to

\[
\dot{r}^2 + V_{\text{eff}}(r) = \mathcal{E}^2,
\]

(26)

where the effective potential has a form

\[
V_{\text{eff}}(r) = f(r) \left[ 1 + \left( \frac{\mathcal{E}}{r} + \omega_B r \right)^2 \right],
\]

(27)

with \( \omega_B = eB/2m \), magnetic coupling parameter (which characterizes the cyclotron frequency of the charged particle in the uniform magnetic field).
3.1 Innermost Stable Circular orbits of charged particle

Now we will consider orbits of charged particles to be circular, or more specifically the innermost stable circular ones. Using the following standard conditions

$$\begin{align*}
V_{\text{eff}}(r) &= \mathcal{E}^2, \\
V'_{\text{eff}}(r) &= 0, \\
V''_{\text{eff}}(r) &= 0,
\end{align*}$$

one can easily find the values of ISCO radius. Angular momentum for circular orbits can also be found from the equations above that reads

$$L_{\pm} = r^2 \left[ \frac{(Mr - Q)\omega_B}{r(r - 3M) + 2Q} \right]$$

One can see from equation (29), clearly that the critical angular moment has two solutions that are marked by signs, it is easy to see that one of them relates to \(\omega_B > 0\) values and the other \(\omega_B < 0\).

$$\mathcal{E} = \frac{Q + (r - 2M)}{r^2} \left[ 1 - \frac{(\omega_B((r - 2M) + Q)}{r(r - 3M) + 2Q} \right]$$

Obviously that in order for the motion to be stable, the angular momentum and energy of the particle must take real value, this satisfies using the following condition:

$$r^2 \omega_B^2 \left( 1 - \frac{2M}{r} + \frac{Q}{r} \right)^2 + Q \left( 1 - \frac{5M}{r} + \frac{2Q}{r^2} \right) - Mr \left( 1 - \frac{3M}{r} \right) \geq 0$$

Since, the first part \(r^2 \omega_B^2 \left( 1 - \frac{2M}{r} + \frac{Q}{r} \right)^2\) is always positive then we have another condition for the motion:

$$Q \left( 1 - \frac{5M}{r} + \frac{2Q}{r^2} \right) - Mr \left( 1 - \frac{3M}{r} \right) \geq 0$$

Equation 32 a little bit complicated show the result in analytically and understand. In order to show the effect of the brane charge effect we show in plot form the condition.

Figure 3 show the dependence of the critical distance where stable circular orbits are allowed on the brane charge. In this figure, we have shown that as an increase in the value of the brane charge the critic distance decreases. One can see from this figure that separated into two "region" by curve line and the top region corresponds
Figure 3: The critical distance for particles around black hole in braneworld.

Figure 4: Radial dependence of the critical angular momentum and orbital energy (at the right panel) of the charged particle in different values of bran charge (the left panel).
to the region where circular stable orbits are allowed, the bottom one corresponds to unstable orbits.

In Figure 4 we illustrated the radial dependence of critical angular momentum for circular orbits (at the right panel of the figure) and particle energy in the circular stable orbits (at the left one). One can see that the maximum values of both the critical angular momentum and the particle energy shift to the central object, as well as the critical distance decrease, and the minimum value of the critical angular momentum for charged particle decreases as the brane charge increases.

\[ \omega_B = 0 \]
\[ |\omega_B| = 0.5 \]

Figure 5: The ISCO radius of the neutral particle as a function of brane charge \( Q \)

Figure 5 shows that the dependence of the ISCO radius of charged particles on the brane charge \( Q \). One can see that the ISCO radius of the charged particle decreases with increasing the value of the magnetic coupling parameter \( |\omega_B| \)

4 Center-of-mass energy of two colliding particles around the black hole in braneworld

In this section, we will study collisions of two particles, in particular, charged-charged and neutral-neutral particles collisions. The expression for the centre-of-mass energy for two-particle system with mass \( m_1 \) and \( m_2 \), in a given gravitational field is as a sum of two-momenta of colliding particles [16]

\[ \{ E_{cm}, 0, 0, 0 \} = m_1 u_1^\mu + m_2 u_2^\mu \]  \quad (33)

where, \( u_1^\mu \) and \( u_2^\mu \) are four-velocity of the two colliding particles and the velocities satisfy the condition \( u_\mu u^\mu = 1 \). Keeping the condition one can square 33 and we have

\[ E_{cm}^2 = m_1^2 + m_2^2 - 2m_1m_2g_{\mu\nu}u_1^\mu u_2^\nu \]  \quad (34)

and after simplifying

\[ \frac{E_{cm}^2}{m_1m_2} = \frac{m_1}{m_2} + \frac{m_2}{m_1} - 2g_{\mu\nu}u_1^\mu u_2^\nu \]  \quad (35)
Now we consider the collision of the charged particles with the same mass \(N = 1\) and initial energy, then the expression for the center-of-mass energy takes the following form using equations of motion charged particles (23).

\[
\mathcal{E}_{cm}^2 = 1 - \left(\frac{l_1}{r} - \omega_B r\right) \left(\frac{l_2}{r} - \omega_B r\right) - \frac{1}{f} + \sqrt{1 + f \left(\frac{l_1}{r} - \omega_B r\right)^2} \left[1 + f \left(\frac{l_2}{r} - \omega_B r\right)^2\right]
\]  

(36)

here \(\mathcal{E} = E/2m\) is dimensionless center-of-mass energy. Now we will study in detail, center-of-mass energy of two colliding (neutral/charged) particles with different cases, i.e. particles with the same mass \(m_1 = m_2 = m\) and different mass \(m_1 \neq m_2\) (assuming \(m_1 = Nm_2\), here \(N\) is some non-zero number) and the angular momentum \((l_1 = -l_2 = l\) it corresponds to head on collision), and initial energies \(\mathcal{E}_1 = \mathcal{E}_2 = 1\) (it means that the colliding particles energy is equal to their rest energy) in the equatorial plane. So, one way analyze the energy is plotting radial dependence of the energy. Here we will study the center-of-mass energy of two colliding charged test particles.

![Figure 6: Radial dependence of center-of-mass energy for different values of brane charge \(Q\) for charged particle with charge \(\omega_B = 1\) and angular momentum \(l = 2\).](image)

We illustrated the center of mass energy as a radial function for different values of the brane charge \(Q\) in Figure 6 for the charged particles with values \(\omega_B = 1\) and the angular values \(l_1 = 2\) and \(l_2 = -2\). One can see from the center of mass energy of the collision is greater when the particles collide near to the horizon and on the horizon the energy reaches its maximum value. Moreover, an increase of the brane charge cause to decrease of center of mass energy. This suggests that the bran charge gives an additional gravitational effect as it obtained in our previous work [13].
Conclusions

In this work, we have considered neutral and charged particle motion as a test particle around the black hole in braneworld. Following important results obtained

- We have studied the circular motion of neutral particle around the black hole in braneworld with $Q > 0$. Shown that the ISCO radius of the particle decreases with increasing the positive values of the charge $Q > 0$
- PW (pseudo-Newtonian) potential for a neutral particle has been considered. It is shown that an increase of the positive value of brane charge cause to increase in the value of PW potential
- Charged particle motion around the black hole immersed in an external asymptotically uniform magnetic field in braneworld has also been studied. The ISCO radius of the charged particle decreases as both the values of magnetic coupling parameter and positive value of brane charge increase.
- We have investigated center-of-mass energy of the charged particles collisions and shown that the brane charge has an additional gravity effect.

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